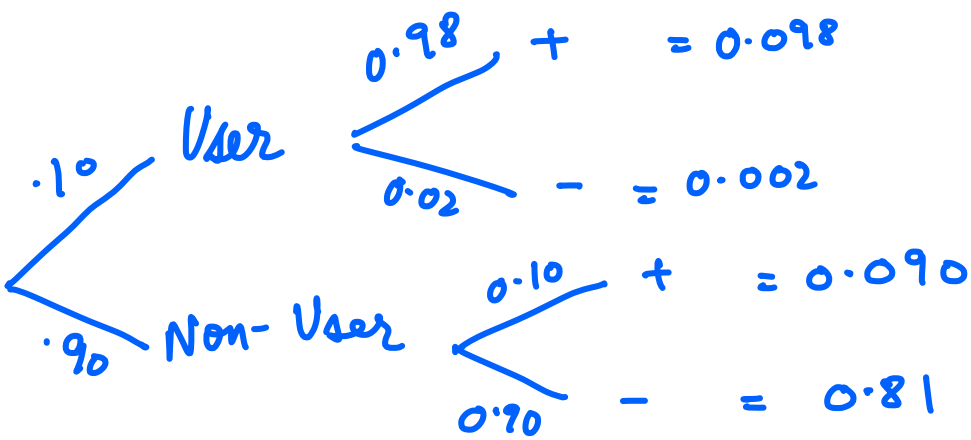
Exam 3 Review

1. (Conditional Probability – Bayes' rule)
Suppose that a drug test for an illegal drug is such that it is 98% accurate in the case of a user of that drug (e.g. it produces a positive result with probability .98 in the case that the tested individual uses the drug) and 90% accurate in the case of a non-user of the drug (e.g. it is negative with probability .9 in the case the person does not use the drug). Suppose it is known that 10% of the entire population uses this drug. You test someone and the test is positive.



a. What is the probability that the individual tests positive for using illegal drug?

$P(+) = P(+ \ \& \ \text{User}) + P(+ \ \& \ \text{Non-user})$

$= 0.098 + 0.090$

$\boxed{= 0.188}$

b. What is the probability that a person who tested positive for this drug is actually a non-user. i.e. Calculate: P(NU | +)

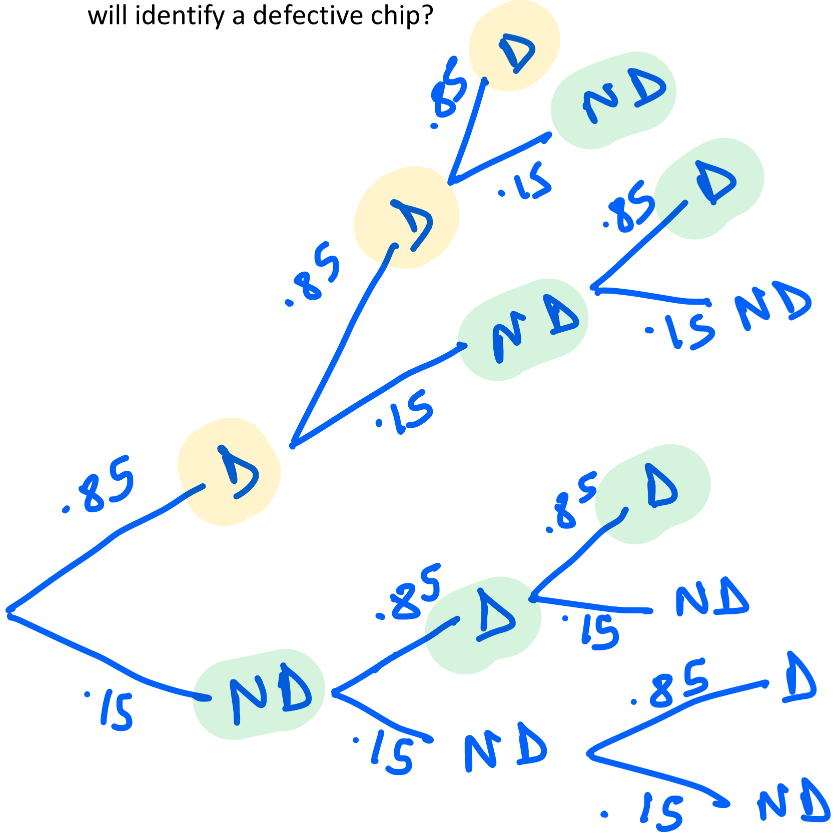
$P(\text{Non-user} \mid +) = \dfrac{P(\text{Non-user} \ \& \ +)}{P(+)} = \dfrac{0.090}{0.188}$

$\boxed{P(NU \mid +) = 0.48}$

2 (tree diagram)

A computer chip manufacturer uses a testing device to determine whether a particular computer chip is defective. This testing device is 85% accurate (i.e. there is 85% chance that a defective computer chip will be identified by such a device).

If each computer chip is being tested by 3 of such devices, what is the probability that at least 2 of them will identify a defective chip?



$P(\text{at least } 2D)$

$= P(2D) + P(3D)$

$= 3 \times (0.85^2 \times 0.15)$

$+ (0.85^3)$

$= 0.93925$

$\simeq 0.94$

3: Suppose it is known that 40% of likely voters in some district intend to vote for a certain candidate. Find a probability that a random poll of 7 people reveals that at least 2 intend to vote for this candidate.

Solution: use calculator (2nd → VARS → binomcdf) to calculate the probability (you should get ≈0.8414)

X: # of people out of 7 who vote for the candidate

$X \sim Bin(n=7,\ p=0.40)$

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - \text{Binomcdf}(7,\ 0.40,\ 1)$

$= 1 - 0.15863$ $\boxed{= 0.84137}$

X: # of people out of 25 who have O-neg blood

$X \sim Bin(n=25, \ p=0.08)$

4. People with blood type O-negative are in higher demand since they are universal blood donors. As of 2012, it was estimated that only 8% of population of Argentina are type O-negative. Suppose 25 people had donated blood to a certain clinic in Argentina last week.

a. What is the probability that at least 3 of these donors were type O-negative?

$$P(X \geq 3) = 1 - P(X \leq 2)$$
$$= 1 - \text{Binomcdf}(25, 0.08, 2)$$
$$= 1 - 0.67684 \ \boxed{= 0.32316}$$

b. What would be the expected number of people out of these 25 donors with type O-negative?

$$E(X) = np = 25 \times 0.08$$
$$\boxed{= 2}$$

c. What is the standard deviation for the number of people with type O-negative blood?

$$\sigma = \sqrt{np(1-p)} = \sqrt{25 \times 0.08 \times 0.92}$$
$$= 1.3565$$

d. Would you be surprised (in other words, "is it unusual") if 7 out of 25 people turn out to be O-negative blood type?

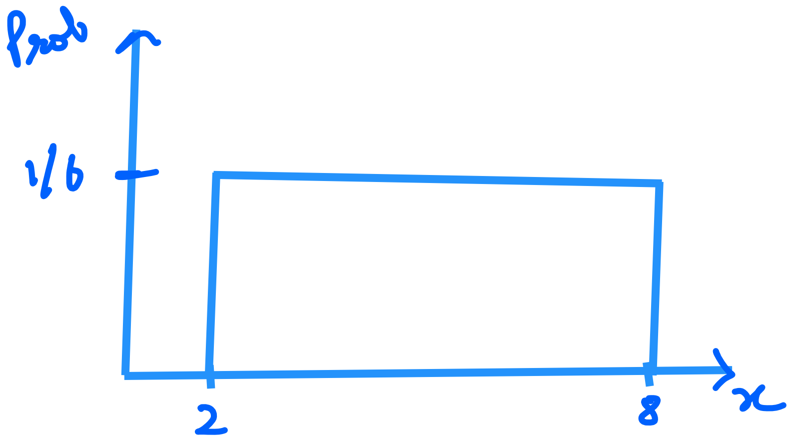
Using the emperical, 95% of the time the # of people with O-ve blood would be in the interval

$$2 \pm (2 \times 1.3565)$$
$$(-0.713, 4.713)$$
$$\text{or } (0, 5)$$

Yes it would be unusual if 7 people had O-ve blood.

5.John eats ice cream every day. The number of ounces consumed by John each day is uniformly distributed between 2 and 8 ounces.

a. What is the height of the probability distribution of the number of ounces of ice cream consumed by John on any given day?



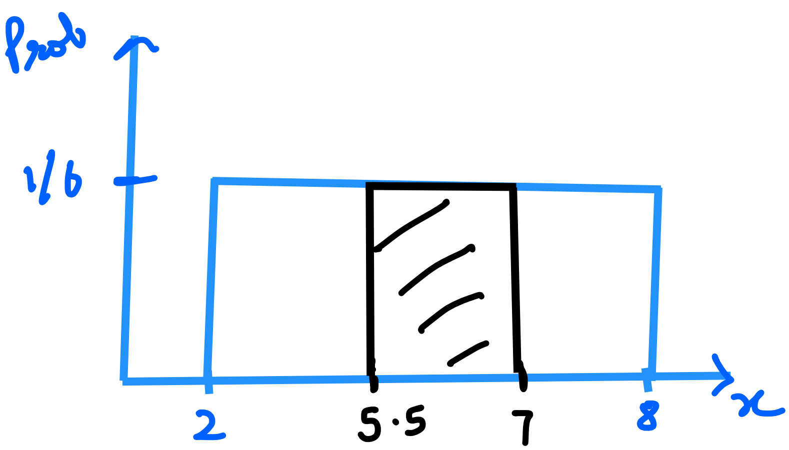
Area of rectangle = 1

$\Rightarrow$ Base $\times$ Height = 1

$6 \times$ Height = 1

$\Rightarrow$ Height = $\frac{1}{6}$

b. What is the probability that John eats between 5.5 and 7 ounces of ice cream tomorrow?



Area of rectangle

= Base $\times$ Height

$= 1.5 \times \frac{1}{6}$

$\boxed{= \frac{1}{4}}$

6.The length in feet of a certain type of snake is normally distributed with a mean of 4.1 feet, and standard deviation of 0.5 feet. What percentage of snakes is expected to be between 3 and 5 feet long?

X: Length (in feet) of snake

$X \sim \text{Normal}(\mu = 4.1,\ \sigma = 0.5)$

$P(3 < X < 5) = \text{normalcdf}(3, 5, 4.1, 0.5)$

$\boxed{= 0.9502}$

X: Avg demand for hotel during Holiday Season

$X \sim \text{Normal}(\mu = 150,\ \sigma = 20)$

7.The average demand for a certain hotel during Holiday Season is normally distributed with a mean of 150 rooms per day and a standard deviation of 20 rooms (per day).

a. Suppose there are 170 rooms available to book for tomorrow. What is the probability that some customers will not be able to get a room for tomorrow?

$P(X > 170) = P(170 < X < \infty)$

$= \text{normalcdf}(170, e99, 150, 20)$

$\boxed{= 0.1587}$

b. How many available rooms does this hotel need to have in order to ensure that all customers will be accommodated with at least 0.99 probability?

$P(X < k) = 0.99$

$\Rightarrow k = \text{invNorm}(0.99, 150, 20, \text{left})$

≈ 197

8.Suppose that a random sample of size 81 is to be selected from a population with mean 30 and standard deviation of 9

a. What is the mean and standard deviation of sample mean: $\bar{X}$?

$n = 81 \qquad \mu_x = 30 \qquad \sigma_x = 9$

$\mu_{\bar{x}} = \mu_x = 30$

$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{9}{\sqrt{81}} = 1$

$\Rightarrow \bar{x} \sim \text{Normal}(\mu = 30, \sigma = 1)$

b. What is the probability that the sample mean for sample size of 81 will be between 28 and 32?

$P(28 < \bar{x} < 32) = \text{Normalcdf}(28, 32, 30, 1)$

$\boxed{= 0.9545}$

c. What is the probability that the sample mean for size of 81 will be greater than 29?

$P(\bar{x} > 29) = P(29 < \bar{x} < \infty)$

$= \text{Normalcdf}(29, e^{99}, 30, 1)$

$\boxed{= 0.8413}$

9.A Gallup Poll asked a sample of Canadian adults if they thought the law should allow doctors to end the life of a patient who is in great pain and near death if the patient makes a request in writing. The poll included 270 people in Quebec, 221 of whom agreed that doctor-assisted suicide should be allowed.

a. Construct a 95% confidence interval for the proportion of all Quebec adults who would allow doctor-assisted suicide?

p : True prop of Quebec adults who would allow doctor-assisted suicide

$n = 270$

$x = 221$

STAT → Tests → 1PropZint (221, 270, 0.95)

= (0.77255, 0.86449)

10. During one holiday season, the Texas lottery played a game called the Stocking Stuffer. The price of a ticket for this lottery was $1.00. Shown below are the various prizes and the probability of winning each prize. To begin complete the probability distribution. Then calculate the expected value of the prize for this game and decide whether it is worthwhile, in the long run, to play. Remember, it costs $1.00 to play!

Prize (X)	**$1000**	**$100**	**$20**	**$10**	**$4**	**$2**	**$0**
Probability	**.00002**	**.00063**	**.00400**	**.00501**	**.03403**	**.14455**	0.81176
X × Prob	0.02	0.063	0.08	0.501	0.13612	0.2891	0

$$E(X) = \sum X \times \text{Prob}$$

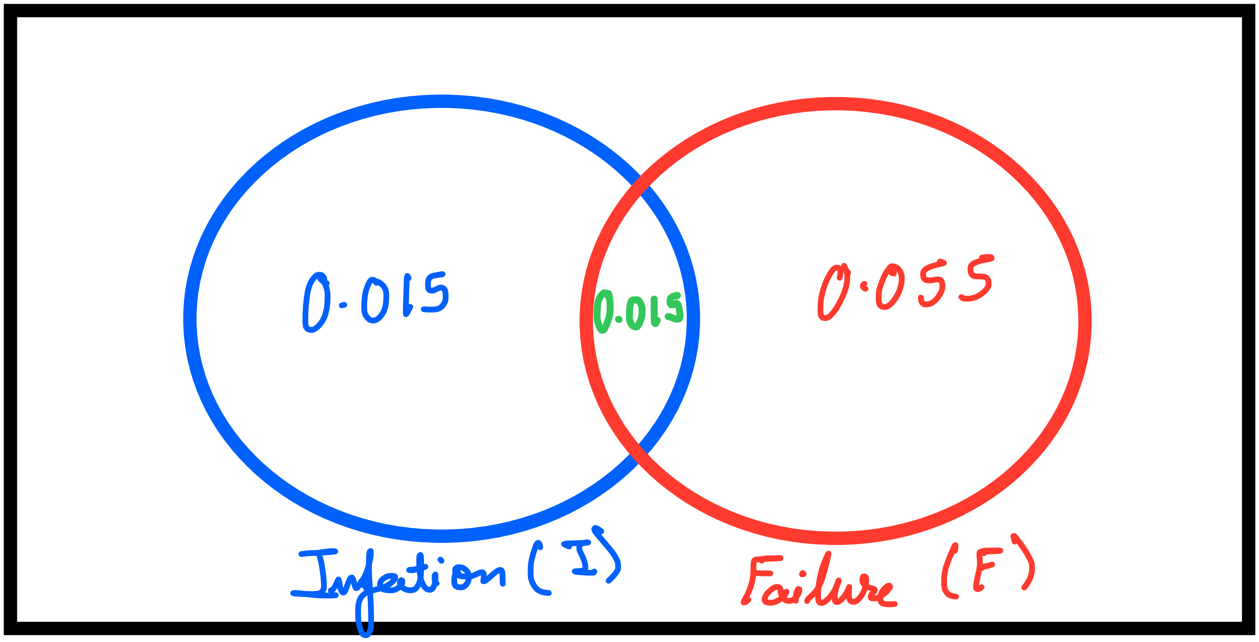
$$= 0.02 + 0.063 + 0.08 + 0.501 + 0.13612 + 0.2891 + 0$$

$$= 1.08922$$

Since it costs $1 to play, $E(\text{Prize}) = 1.08922 - 1$

$$= 0.08922$$

11. An elderly individual has a fractured hip and is facing surgery to repair it. The orthopedic surgeon explains the risks to the patient. Infection occurs in 3% of such operations, the repair fails in 7%, and both infection and failure together in1.5%. (Use a Venn Diagram to model this problem.)



a. What is the probability that the operation succeeds and is free from infection?

$$P(\text{Op succeeds \& No Infection}) = P(F^c \cap I^c)$$
$$= 1 - (0.015 + 0.015 + 0.055)$$
$$= 1 - 0.085 = \boxed{0.915}$$

b. What is the probability that the repair fails, given that an infection occurs?

$$P(F \mid I) = \frac{P(F \& I)}{P(I)} = \frac{0.015}{0.03} \boxed{= 0.5}$$