Question 1

a) Use the one-to-one property to solve the equation for x.

$$\left(\frac{1}{2}\right)^x = 32$$

$$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-5}$$

$$\underline{x = -5}$$

b) Describe the transformation(s) of the graph of f that yield(s) the graph of g.

$$f(x) = 10^x, \quad g(x) = 10^{-x+3}$$

reflection along the y-axis

Shift 3 units left

Question 2

Evaluate the logarithm at the given values of x without using a calculator.

$f(x) = \log_8 x$	$x = 1$

$f(1) = \log_8 1 = \underline{0}$

Since $8^0 = 1$

$g(x) = \log_a x$	$x = a^{-2}$

$g(a^{-2}) = \log_a a^{-2} = \underline{-2}$

Since $a^{-2} = a^{-2}$

Question 3

Find the domain, x-intercept, and vertical asymptote of the logarithmic function and sketch its graph.

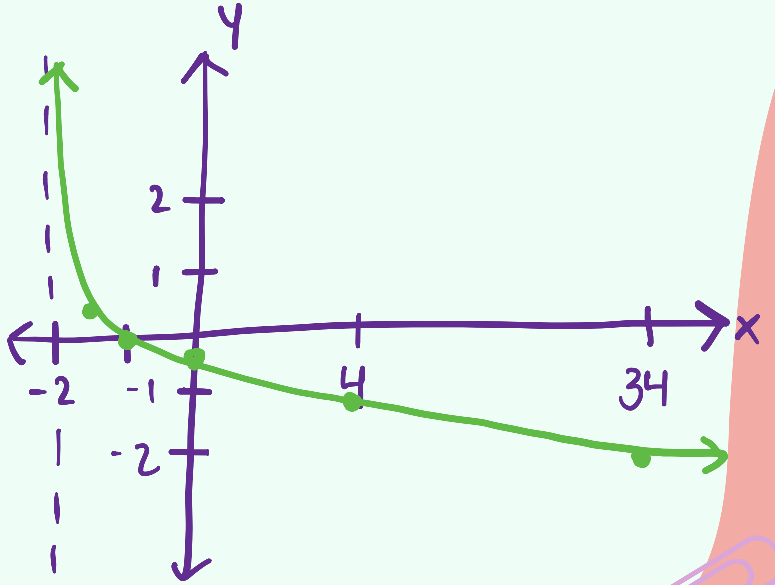
x	$f(x)$
$-3/2$	0.387
-1	0
0	-0.387
4	-1
34	-2

$$f(x) = -\log_6(x+2)$$

Domain: $(-2, \infty)$

x-intercept: $(-1, 0)$

VA: $x = -2$



Question 4

a) Use the properties of logarithms to expand the expression.

$$\log_5 \frac{x^2}{y^2 z^3}$$

$\log_5 x^2 - \log_5 y^2 z^3$

$\log_5 x^2 - [\log_5 y^2 + \log_5 z^3]$

$\log_5 x^2 - \log_5 y^2 - \log_5 z^3$

$\underline{2\log_5 x - 2\log_5 y - 3\log_5 z}$

b) Use the properties of logarithms to condense the expression.

$$\tfrac{1}{3}[\log_8 y + 2\log_8(y+4)] - \log_8(y-1)$$

$\frac{1}{3}\log_8 y + \frac{2}{3}\log_8(y+4) - \log_8(y-1)$

$\log_8 y^{1/3} + \log_8 (y+4)^{2/3} - \log_8(y-1)$

$\log_8 y^{1/3}(y+4)^{2/3} - \log_8(y-1)$

$\log_8 \frac{y^{1/3}(y+4)^{2/3}}{(y-1)} = \underline{\log_8 \frac{\sqrt[3]{y(y+4)^2}}{y-1}}$

Question 5

a) Solve the <u>exponential</u> equation algebraically.

$$6(2^{3x-1}) - 7 = 9$$

$$6(2^{3x-1}) = 16$$

$$2^{3x-1} = 16/6 = 8/3$$

$$\log_2 2^{3x-1} = \log_2 8/3$$

$$3x - 1 = \log_2 8/3$$

$$3x = \log_2 8/3 + 1$$

$$x = \tfrac{1}{3}(\log_2 8/3 + 1) \approx 0.805$$

b) Solve the <u>logarithmic</u> equation algebraically.

$$\ln(x+5) = \ln(x-1) - \ln(x+1)$$

$$\ln(x+5) = \ln\left(\frac{x-1}{x+1}\right)$$

$$e^{\ln(x+5)} = e^{\ln\left(\frac{x-1}{x+1}\right)}$$

$$x+5 = \frac{x-1}{x+1}$$

$$(x+5)(x+1) = x-1$$

$$x^2 + 6x + 5 = x - 1$$

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

$$x = -3, -2$$

Check solutions!

Because you get the natural log of a negative number, the value is undefined $\Rightarrow$ $\boxed{\text{No solution}}$

Question 6

a) Find the missing values assuming continuously compounded interest.

Initial Investment	Annual % Rate	Time to Double	Amount After 10 Years
	4.5%		$10,000.00

Initial Investment :

$A = Pe^{rt}$

$10{,}000 = Pe^{0.045(10)}$

$P = 10{,}000 / e^{0.045(10)}$

$\approx \underline{\$6{,}376.28}$

Time to Double :

$A = 2P = Pe^{rt}$

$2 = e^{rt} = e^{0.045t}$

$\ln 2 = \ln e^{0.045t}$

$\ln 2 = 0.045t \Rightarrow t = \frac{\ln 2}{0.045} \approx \underline{15.4 \text{ years}}$

b) Find the missing values for the radioactive isotope.

Isotope	Half-life (years)	Initial Quantity	Amount After 1000 Years
^{14}C	5715		2 g

Initial Quantity :

$f(t) = a\left(\frac{1}{2}\right)^{t/h}$

remains of material

initial amount

time

half-life time

$2 = a\left(\frac{1}{2}\right)^{1000/5715}$

$a = 2 / \left(\frac{1}{2}\right)^{1000/5715}$

$\approx \underline{2.26 \text{ g}}$

Question 7

Find (if possible) the complement and supplement of each angle.

(a) $\frac{\pi}{12}$ (b) $\frac{11\pi}{12}$

(a) Complement: $\frac{\pi}{2} - \frac{\pi}{12} = \frac{6\pi}{12} - \frac{\pi}{12}$

$= \underline{5\pi/2}$

Supplement: $\pi - \pi/12 = \frac{12\pi}{12} - \pi/12$

$= \underline{\frac{11\pi}{2}}$

(b) Complement: There is <u>no complement</u> of $11\pi/12$ because $11\pi/12$ is greater than $\pi/2$.

Supplement: $\pi - 11\pi/12 = \frac{12\pi}{12} - \frac{11\pi}{12}$

$= \underline{\pi/12}$

Question 8

Convert each degree measure to radian measure as a multiple of π. Do not use a calculator.

(a) $120°$ (b) $-20°$

(a) $120° \cdot \dfrac{\pi \text{ rad}}{180°} = \underline{\dfrac{2\pi}{3} \text{ rad}}$

(b) $-20° \cdot \dfrac{\pi \text{ rad}}{180°} = \underline{-\dfrac{\pi}{9} \text{ rad}}$

Question 9

Find the length of the arc on a circle of radius r intercepted by a central angle θ.

$r = 15$ inches, $\theta = 120°$

Step 1 : convert degrees to radians

$$120° \cdot \frac{\pi \text{ rad}}{180°} = \frac{2\pi}{3} \text{ rad}$$

Step 2 : use arc length formula

$$s = r\theta$$
$$= 15\left(2\pi/3\right)$$
$$= 10\pi$$
$$\approx 31.42 \text{ in.}$$

Question 10

a) Evaluate (if possible) the six trigonometric functions at the real number.

$t = -5\pi/3$ $= \pi/3$

$\sin(\pi/3) = \underline{\frac{\sqrt{3}}{2}}$

$\cos(\pi/3) = \underline{1/2}$

$\tan(\pi/3) = \frac{\sqrt{3}/2}{1/2} = \underline{\sqrt{3}}$

$\csc(\pi/3) = \frac{2}{\sqrt{3}} = \underline{\frac{2\sqrt{3}}{3}}$

$\sec(\pi/3) = \underline{2}$

$\cot(\pi/3) = \frac{1/2}{\sqrt{3}/2} = 1/\sqrt{3} = \underline{\sqrt{3}/3}$

b) Evaluate the trigonometric function using its period as an aid.

$\cos(7\pi/3)$

$\cos\left(\frac{7\pi}{3}\right) = \cos\left(\pi/3 + 6\pi/3\right)$

$= \cos(\pi/3 + 2\pi)$

$= \cos(\pi/3)$

$= \underline{1/2}$